The Effect of Pre-Engineering Activities on 4th and 5th Grade Students’ Understanding of Rectangular Solids Made of Small Cubes

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ABSTRACT

The purpose of this research was to investigate the effect of pre-engineering activities on 4th- and 5th-grade students understanding of rectangular solids made of small cubes. The study utilized a pretest-posttest experimental design with 121 students. During a two class-hour instruction, experimental groups constructed simple toys such as cars, ships, and trucks out of wooden cubes and triangular prisms. Control groups continued on their regular in-class activities. Results showed that experimental groups, with no gender differences, statistically significantly improved their understanding of three-dimensionality and spatial structuring of three-dimensional arrays of cubes. Similar activities can be used in classrooms to help students mentally construct three-dimensionality and improve their understanding of the spatial structure of rectangular buildings made of small cubes.

Key Words: three-dimensionality, rectangular buildings, spatial structuring, volume

Introduction

Many studies have shown that children had difficulty finding the number of cubes in rectangular solids even in high school (Battista & Clements, 1996; Ben-Chaim, Lappan, & Houang, 1985) and beyond (Hirstein, 1981). Finding the number of cubes in rectangular solids provides the cognitive framework for understanding the measurement of volume and the formula for determining the volume (Battista & Clements, 1998). Without a proper understanding of the spatial structure of these buildings, students think of the volume formula as just a product of three numbers.

For any mathematical concept such as volume, students pass through various levels of understanding. Mathematics education researchers attempt to clarify what these levels are and how these levels of understanding are attained as well as to study what to do to help students progress from a primitive conceptualization to a more sophisticated understanding of the concept. Previously, researchers have described students’ difficulties and the cognitive constructions students made as they enumerated 3-D arrays of cubes (Battista & Clements, 1996). Further studies (e.g., Olkun, 2003) confirmed the developmental trajectories of students in similar tasks.

Based on findings that students counted the faces of cubes or visible cubes while they were finding the number of cubes in rectangular buildings, Hirstein (1981) claimed that students confused the volume with the surface area. Ben-Chaim, Houang, and Lappan (1985) found that students sometimes counted the cubes on the edges or corners of prisms twice or more than twice. They asserted that these students had difficulty in visualising the pictorial representations of 3-D buildings; they did not use concrete materials in their study. Battista and Clements (1996) found that students used similar strategies in finding the number of cubes in rectangular buildings made of small cubes for both concrete and pictorial situations. Contrary to the assertions of Ben-Chaim, Houang and
Lappan, the latter (Battista and Clements) attributed the students’ mistakes to inappropriate spatial structuring of the rectangular arrays of cubes.

Battista and Clements (1996) defined spatial structuring as “constructing units, forming composite units such as columns and layers by iterating units, and finally constructing the whole building by iterating composite units” (p. 282). How students structured or conceptualized the building can be inferred from the strategy they used while enumerating the cubes in the building. Students’ conceptualizations are mainly of three types (see Table 1). These three conceptualizations are labelled C, B, and A for ease of depiction.

Table 1. Students’ Conceptualizations of Rectangular Arrays of Cubes

<table>
<thead>
<tr>
<th>Type</th>
<th>Conceptualization</th>
<th>Unit used</th>
<th>Whole structuring</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>A set of faces</td>
<td>Cube faces</td>
<td>Based on faces</td>
</tr>
<tr>
<td>B</td>
<td>A set of cubes</td>
<td>Individual cubes</td>
<td>Partial or local</td>
</tr>
<tr>
<td>A</td>
<td>Organized cubes</td>
<td>Cubes, rows, columns and layers</td>
<td>Holistic</td>
</tr>
</tbody>
</table>

Students with “C” conceptualization act on buildings based on faces. They take individual cube faces as units and their overall structuring is based on building’s faces. They do not consider the drawing as three-dimensional nor do they consider the interior cubes in the concrete building. As a result, they treat the building as a set of faces. Students with “B” conceptualization are aware of the three-dimensionality and space-filling properties of the cubes and the whole building. They use cubes as units but their overall structuring is local, not yet global. For them, the building is a “bunch of cubes.” They usually count the cubes one by one and unsystematically. On the other hand, students with "A" conceptualization utilize composite units or units of units and unit iteration. For them, the cube building is organized into regular patterns such as columns, rows, and layers.

Students may not use the same strategy for all kinds of rectangular buildings. In fact, students return to a primitive strategy when they are presented a more complex building. Therefore, the complexity of the building at hand has an effect on the strategies they choose (Olkun, 1999). The classical pedagogic principle that students move from concrete to pictorial and to symbolic representation may be utilized in designing instructional activities that could lead students gain a deeper understanding of volume-related concepts and skills.

Understanding the 2-D representations of 3-D buildings is a part of spatial visualization. Spatial visualization includes mental integration of different views, such as orthogonal views, into a whole body. Battista and Clements (1998) reported that elementary students are unable to coordinate the different orthogonal views of the cube configuration. In fact, to construct the 3-D cube building correctly and to explore the invisible cubes, mental configuration of the orthogonal views is necessary. Cohen (1979) stated that teachers should provide appropriate classroom environment where students can visualize the different views of constructions, match the objects with their pictures, and observe them with their different perspectives. These skills are very much like the ones used in engineering activities such as reading drawings of buildings and producing parts out of some materials.

Many studies have shown that there are considerable gender differences on spatial visualization tests. Generally boys are more successful than girls. Strong and Smith (2002) claimed that these differences come from the different kinds of child’s play. Similarly, Baenninger and Newcombe (1989) found a reliable relationship between spatial activity participation and spatial ability. The more a subject had participated in spatial activities such as playing with blocks and drawing in three dimensions, the higher his/her spatial visualization score. Traditionally, boys have more chance to
participate in spatial activities than girls do (Robichaux, 2002). Engaging girls in additional construction activities may help them improve their spatial skills and close the gap with boys.

The present study aims at extending previous research about students’ understanding of rectangular solids made of small cubes and attempts to extend the research base by utilizing different problem situations, such as an engineering context for constructing 3-D buildings. Specifically, the purpose of this research was to investigate the effect of pre-engineering activities on fourth- and fifth-grade students’ understanding of rectangular solids made of small cubes.

**Methods**

**Participants**

A total of 121 students (62 fourth graders and 59 fifth graders) participated in the study. The school from which the participants were drawn was located in a mixed socioeconomic area in Ankara province, the capitol of Turkey. The students voluntarily participated in the study. They said they had not previously seen the materials used in the study.

**Materials**

The test items consisted of six open-ended questions, which required students to find the number of cubes in the rectangular buildings represented pictorially (see Figure 1). The six buildings ranged from a one-layer building (2 by 2 by 1) to a complex one (3 by 4 by 5 building). Such questions can be found in large-scale mathematics achievement tests such as NAEP (National Assessment of Educational Progress) as well as in small-scale research studies (Olkun, 2003; 1999; Battista & Clements, 1996; Ben-Chaim, Lappan, & Houang, 1985). The reanalysis of the test instruments reliability revealed a sufficiently high coefficient (alpha = 0.84, N = 149, number of items = 6). The same test was used for pretesting and posttesting.

Instructional materials included small wooden cubes and triangular prisms. The pictorial representations of toys (see Figure 2) were also present during the instruction. There were 11 activities in which students were first asked to predict the number of cubes needed to construct the toys out of identical wooden cubes and triangular prisms by looking at their pictorial representations, which included both orthographic and perspective drawings of toys. Then they were asked to actually construct the buildings using wooden cubes and triangular prisms. These 11 activities were specially developed from a very simple structure to a more complex one. No rectangular buildings (as the ones used for testing) were included in the instructional tasks.

![Image of instructional materials](image_url)

This is a unit cube. How many unit cubes are there in this building? Explain how you found ……………………………

How many unit cubes are there in this building? Explain how you found. ……………………………

**Figure 1.** Two tasks (one simple and one complex) from the testing instrument

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Procedure

Instructional materials were piloted with a small group of fourth- and fifth-grade students before being used for the instruction. After necessary revisions, all fourth and fifth graders of the school were pretested. Fourth- and fifth-grade students were separately assigned in control and in experimental groups. Before the instruction, there were no statistically significant differences between the groups based on their pretest scores. During a two-hour instruction, experimental groups constructed such toys as cars, ships, and trucks out of wooden cubes and triangular prisms by looking at their pictorial representations that included both orthographic and perspective drawings of buildings (see Figure 2). Control groups continued on their regular in-class activities. A week after the instruction, both control and experimental groups were posttested in separate rooms within the same day.

How many unit cubes and triangular prisms do you think were used to construct the toy shown below?

Write your predictions first.

Unit cube: .............

Triangular prism: .............

The arrows indicate the directions from which the toy was looked at. How the toy was seen from these directions is given in the same picture.

Find the direction for each view and write in the callouts.

Now, construct the toy out of wooden cubes and triangular prisms. Then, find out how many cubes and triangular prisms you used and write them down.

Unit cube: .............

Triangular prism: .............

Are your predictions and actual numbers the same or different? .....................................

Figure 2. A sample activity from the instructional material

Results and Discussion
The pretest and posttest scores of students in the experimental and control groups are presented in Table 2. As seen from the table, there were no statistically significant differences between control and experimental groups’ pretest scores while there was a statistically significant difference after the instruction. We can confidently say that experimental groups benefited from the instructional activities.

**Table 2. Pretest and Posttest Scores of Control and Experimental Groups**

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>X</th>
<th>SD</th>
<th>t</th>
<th>X</th>
<th>SD</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>51</td>
<td>2.06</td>
<td>1.881</td>
<td>.127</td>
<td>2.35</td>
<td>2.057</td>
<td>4.061***</td>
</tr>
<tr>
<td>Experimental</td>
<td>70</td>
<td>2.01</td>
<td>1.930</td>
<td></td>
<td>3.70</td>
<td>1.591</td>
<td></td>
</tr>
</tbody>
</table>

***p<0.001 (Maximum score was 6.)

Table 3 depicts the students’ pretest and posttest scores, standard deviations, and t values at grade level. It is evident from the table that the mean scores of each grade level are very close to each other based on their pretest scores. Their posttest scores, on the other hand, show that experimental groups got statistically significantly higher scores than the control groups.

These findings indicate that students benefited from the two class-hour instructional intervention in which students constructed toys with identical wooden cubes and triangular prisms. In order to reveal whether this improvement in students’ performance is consistent across students and test items, the 10 most improved student’s responses to test items were examined.

**Table 3. Students’ Pretest and Posttest Scores According to Grade Levels**

<table>
<thead>
<tr>
<th>Grade</th>
<th>N</th>
<th>X</th>
<th>SD</th>
<th>t</th>
<th>X</th>
<th>SD</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>4(cont.)</td>
<td>26</td>
<td>1.92</td>
<td>1.623</td>
<td>.135</td>
<td>2.46</td>
<td>1.86</td>
<td>3.238**</td>
</tr>
<tr>
<td>4(exp.)</td>
<td>36</td>
<td>1.86</td>
<td>1.885</td>
<td></td>
<td>3.89</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>5(cont.)</td>
<td>25</td>
<td>2.20</td>
<td>2.141</td>
<td>.043</td>
<td>2.24</td>
<td>2.278</td>
<td>2.376*</td>
</tr>
<tr>
<td>5(exp.)</td>
<td>34</td>
<td>2.18</td>
<td>1.992</td>
<td></td>
<td>3.50</td>
<td>1.581</td>
<td></td>
</tr>
</tbody>
</table>

** p < 0.01; * p < 0.05  (Maximum score was 6.)

Table 4 illustrates the patterns of the 10 most improved students’ responses to test items. As seen from the table, students’ responses show a linear improvement from simple to more complex items and this improvement is fairly consistent. In addition, students in the experimental groups increased their number of correct answers for simple items more often than they did for complex items. This finding further shows that students had started to spatially structure rectangular buildings in their minds for simple buildings but not for more complex buildings yet.
Some students either invented or imposed a multiplicative strategy to enumerate the number of cubes for a large building on the posttest while they were using the “count the faces” strategy for the same building, which is a much less viable strategy (see Figure 3). This improvement in strategy use shows that they started to see the buildings in terms of layers and not as a set of faces anymore.

Table 4. The Patterns of Students’ Wrong and Correct Responses to Test Items

<table>
<thead>
<tr>
<th></th>
<th>PRETEST</th>
<th>POSTTEST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
</tr>
<tr>
<td>4th Grade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taha</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Özgün</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Elif</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tugay</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ezgi</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5th Grade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mert</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Akin</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Süme</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Özgün</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Tümay</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Q: Question
Pretest

Name: Ayşe...

Number of cubes: 36...

Name: Ali...

Number of cubes: 44...

Name: Ümit...

Number of cubes: 41...

Posttest

Name: Ayşe...

Number of cubes: 60...

Name: Ali...

Number of cubes: 60...

Name: Ümit...

Number of cubes: 60...

Figure 3. Changes in student strategies
Gender differences

When the experimental groups were selected, there were no statistically significant differences between fourth-grade boys’ and girls’ scores both on pretest (N = 36, t = 1.052, p = .300) and posttest (N = 36, t = .397, p = .694). Similarly, there were no statistically significant differences between fifth-grade boys’ and girls’ scores before the instruction (N = 34, t = 1.777, p = .085) however, boys performed statistically significantly better than the girls on the posttest (N = 34, t = 2.174, p = .037). In fact, all groups were improved (see Table 5 for details) but girls improved relatively more, especially fourth graders. This might have been caused by the fact that girls had a larger room for improvement since they had lower pretest scores. However, boys scored consistently higher than girls both on pretest and posttest.

Table 5. Paired T-test Results of Boys and Girls

<table>
<thead>
<tr>
<th>GRADE</th>
<th>Gender</th>
<th>N</th>
<th>PRETEST</th>
<th>POSTTEST</th>
<th>GAIN SCORE</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Girls</td>
<td>11</td>
<td>1.36</td>
<td>3.73</td>
<td>2.36</td>
<td>5.004**</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>25</td>
<td>2.08</td>
<td>3.96</td>
<td>1.88</td>
<td>4.784**</td>
</tr>
<tr>
<td>5</td>
<td>Girls</td>
<td>17</td>
<td>1.59</td>
<td>2.94</td>
<td>1.35</td>
<td>3.533**</td>
</tr>
<tr>
<td></td>
<td>Boys</td>
<td>17</td>
<td>2.76</td>
<td>4.06</td>
<td>1.29</td>
<td>2.814*</td>
</tr>
</tbody>
</table>

** p<0.01; * p<0.05   (Maximum score was 6.+)

Conclusions and Implications

Results show that all students benefited from the two-hour instructional activities. This finding is very consistent across grades and genders. Considering that toys students constructed out of identical wooden cubes and triangular prisms are generally used among boys, the improvement girls made with the same activities is especially noteworthy. Regardless of the types of toys constructed, we can say that such construction activities in an engineering context are very helpful in improving students’ understanding of rectangular solids made of small cubes. Improvement in students’ understanding of rectangular solids made of small cubes was evident from their scores on both tests as well as from the strategies they used.

The attempts students made in constructing toys out of identical blocks by looking at their pictorial representations, trying to figure out how many blocks were used for the construction, might have enabled students to discover the spatial relationships between the elements of buildings as well as the relationship between the concrete building and its pictorial representation. Pictorial representations included both perspective and orthographic views. During the instruction, students asked many questions about orthographic views and had difficulty associating those views with the perspectives. It was evident from the students’ questions that perspective drawings were more easily understood than orthographic projections. At the beginning, they treated the orthographic views as separate buildings with no depth. After some time, however, they tried to check their buildings by looking at them from different sides.

During the construction, they internalized the ordered structure of buildings. While they were trying to construct the toy buildings, they realized that there were cubes that were not visible and these also need to be counted. For example, to account for the invisible cubes at a lower layer, they removed the top layer and saw that indeed, there were cubes beneath the top layer. Having perceptual inputs from the concrete buildings and constantly comparing different views helped the
students overcome their difficulties. At the end of the instruction, there was no such difficulty at least for relatively simpler buildings. They implicitly coordinated and integrated different views of cubes and cube buildings through their constructing actions (Battista & Clements, 1996). They were able to account for more of the invisible cubes in the posttest in which they were not allowed to remove the layers since they were presented only pictorial representations of buildings.

The findings in Table 4 especially show that students’ improvement is very solid and consistent considering the uniformity of correct answers to the same test items. Regardless of gender, all students benefited from the activities. These activities are proven to be helpful before introducing the students to the volume formula. In fact, such activities may lead students to discover the volume formula by themselves. Further research may concentrate on how students go about discovering the volume formula.

References


